Connectivity Preserving Iterative Compression

When applying iterative compression to solve an optimization problem, we construct a smaller auxiliary graph from the input graph, solve the problem on this smaller graph, and then use the solution to solve the original problem.

We present a variant of this approach in which the smaller graph inherits the connectivity properties of the original graph. This is especially useful when trying to solve embedding or routing problems. We present three applications which illustrate this.

To warm up we present a simple linear time algorithm which tests planarity and embeds a planar input in the plane. We then present, for every fixed k, a linear time algorithm which determines if an input graph has crossing number at most k, and obtains a corresponding embedding if it does. Finally we present a linear time algorithm which given four vertices s_1,s_2,t_1,t_2 of a graph G determines if there are 2 vertex disjoint paths P_1 and P_2 such that P_i contains s_i and t_i. It either finds the desired two paths or an embedding *up to 3-cuts* of G in a disc which proves that the paths cannot exist.

As well as introducing the new technique, our talk will highlight some of the connections between graph minor theory and embedding algorithms.

(joint work with Kenichi Kawarabayashi and Zhentao Li).